Numerical Optimization in Support to Graph-based Scenario Modelling

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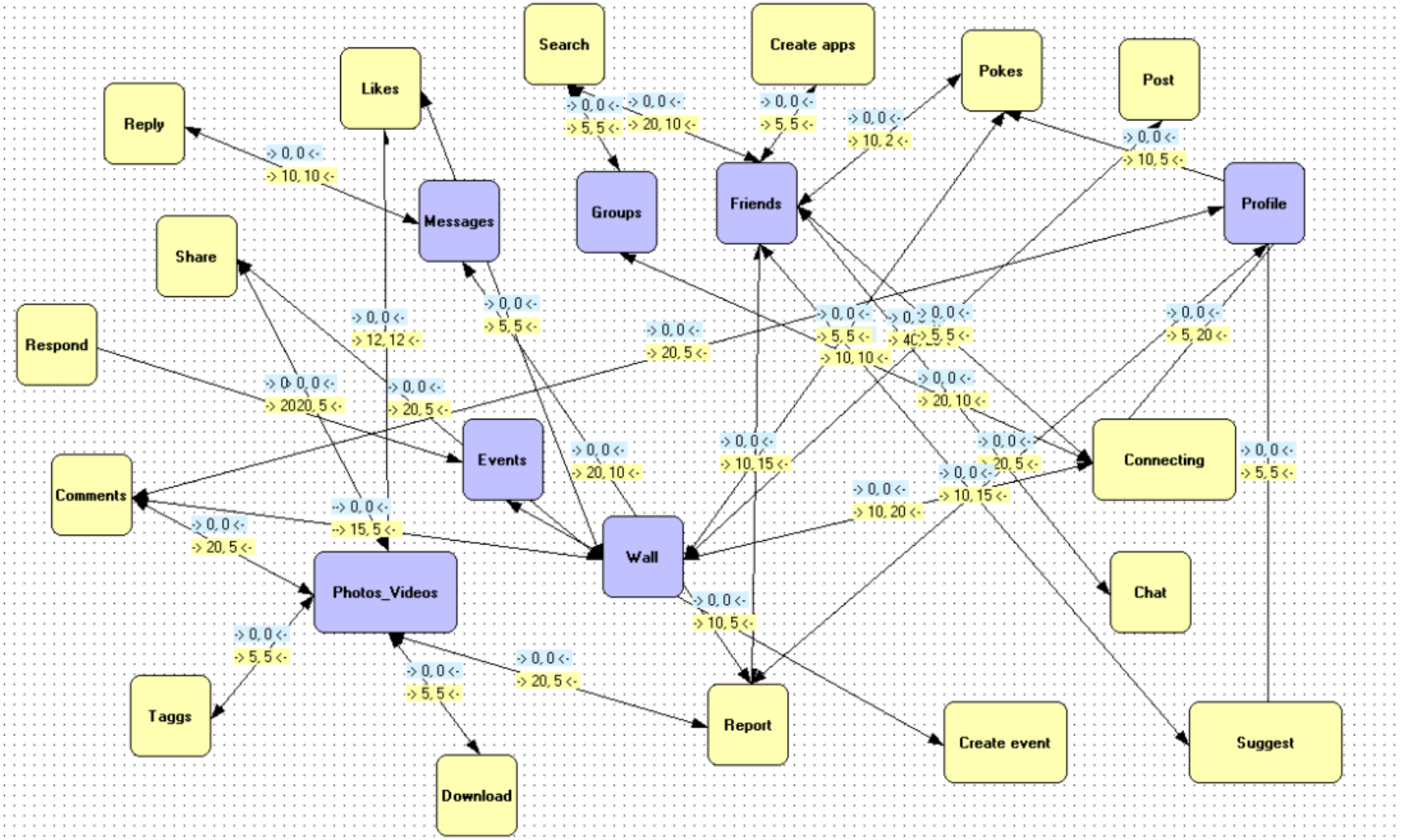
The Model*

I-SCIP®



*Minchev & Petkova, “Information Processes and Threats in Social Networks: A Case Study”, 2010.

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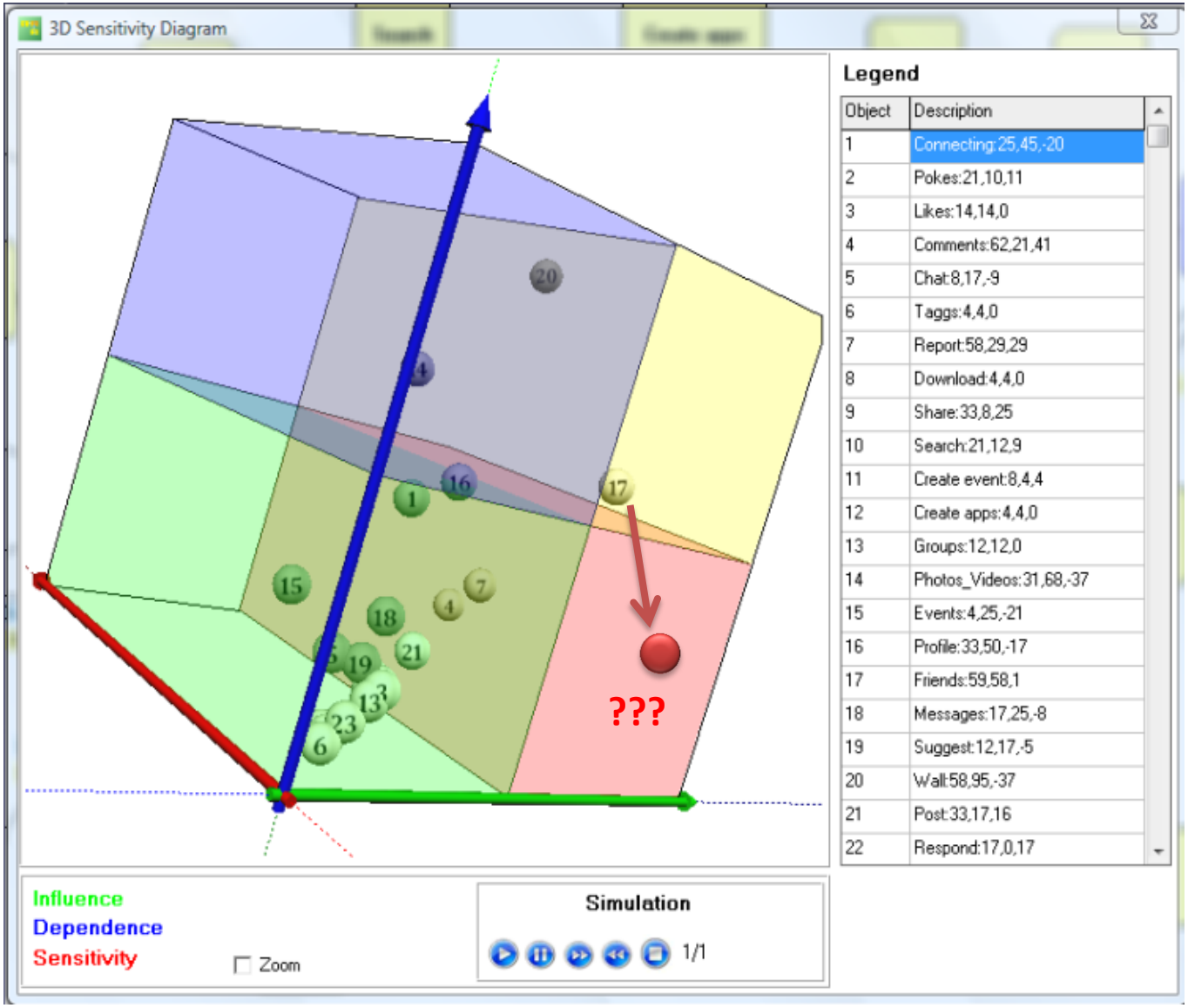
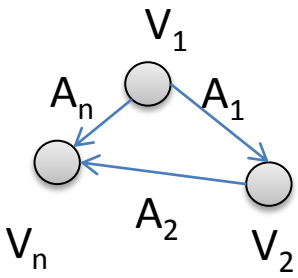
The Task



**Moving the objects in-between the four cluster zones (Red, Green, Blue, Yellow) defined in accordance with the Influence/Dependence values measured in percentages [0,100] from the interval [0,1] in I-SCIP program Sensitivity Diagram - SD.*

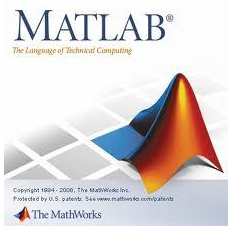
An Algebraic Interpretation & Quadratic Optimization Usage

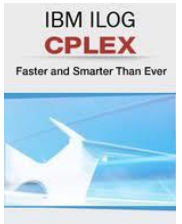
Directed Weighted Graph $G = (V,A)$



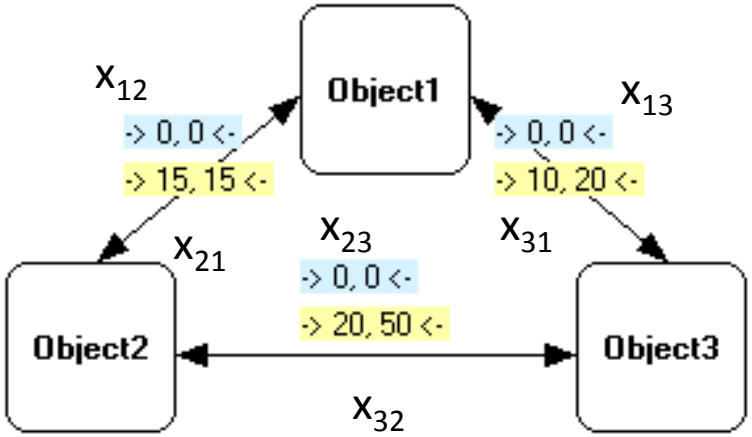
$A = \{A_1, A_2, \ldots, q_i, \ldots, A_n\}$ with Q weights, where $Q = \{q_1, q_2, \ldots q_i, \ldots, q_n\}$, $q \in N$, $q \in [1, 100]$

$Z = (\Sigma q_i - \alpha)^2 + (\Sigma p_j - \beta)^2$,
s.t. $0 < \Sigma q_i \leq \alpha$, $0 < \Sigma p_j \leq \beta$
$i=1,\ldots,n$, $j=1,\ldots,m$; α, β - desired position in the cluster set
Minimize $\rightarrow Z$





Example



Minimize the Objective Function Z:

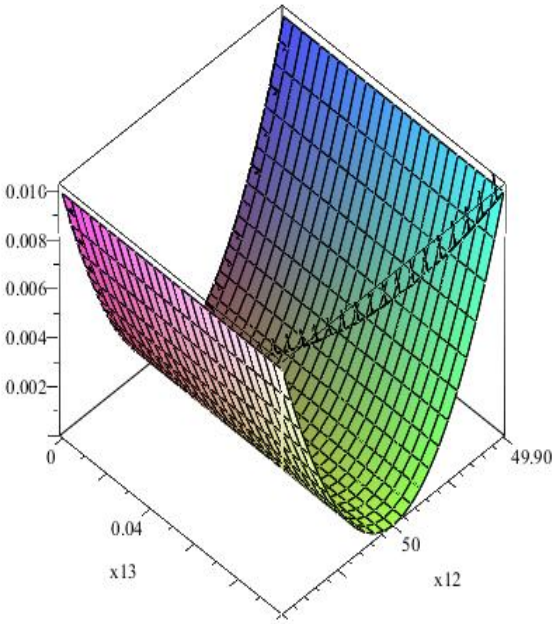
$$(x12 + x32 - 65)^2 + (x21 + x23 - 80)^2$$

S.t. the following constraints:

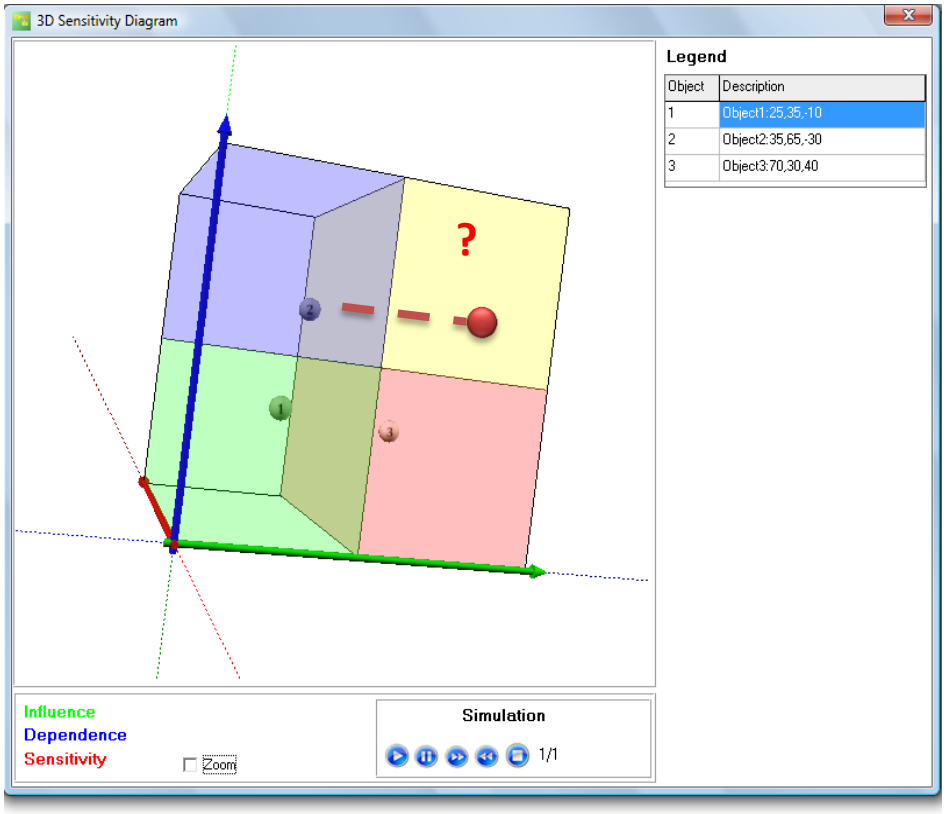
$$x12 \in [0, \infty)$$
$$x13 \in [0, \infty)$$
$$x21 \in [0, \infty)$$
$$x23 \in [0, \infty)$$
$$x31 \in [0, \infty)$$
$$x32 \in [0, \infty)$$
$$x21 + x31 \leq 50$$
$$0 \leq x21 + x31$$
$$x12 + x13 \leq 50$$
$$0 \leq x12 + x13$$
$$x13 + x23 \leq 50$$
$$0 \leq x13 + x23$$
$$x31 + x32 \leq 50$$
$$0 \leq x31 + x32$$

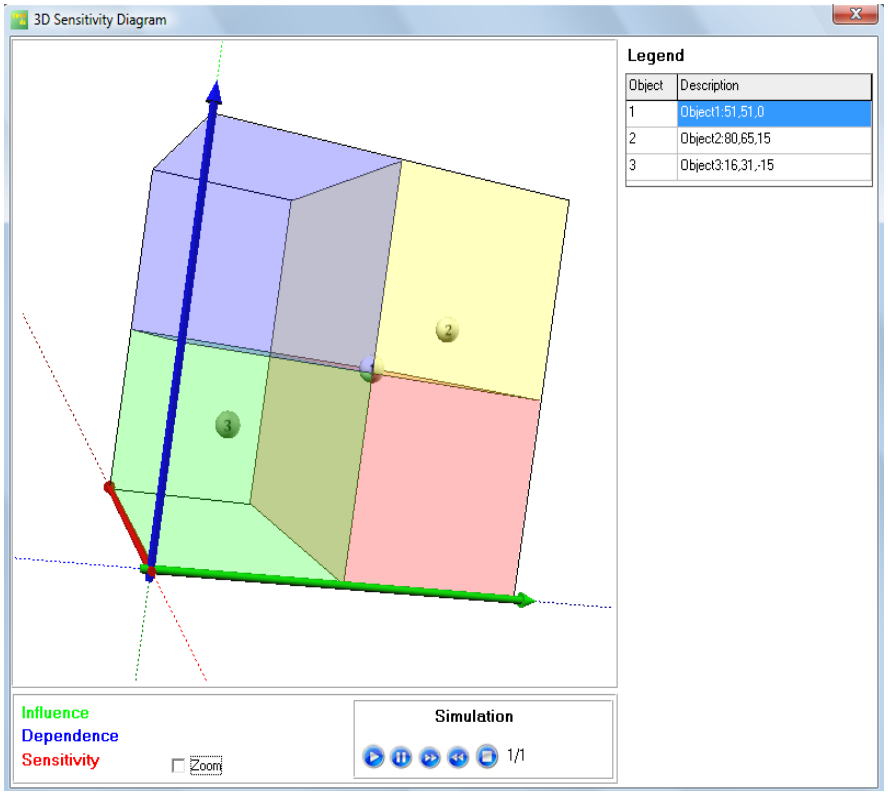
Solution:

The following warning was issued while solving:
necessary conditions met but sufficient conditions not satisfied
Objective value: 0.
x12 = 50. x13 = 0. x21 = 50. x23 = 30.
x31 = 0. x32 = 15.



But what does this mean in the clusters' sense for I-SCIP SD?

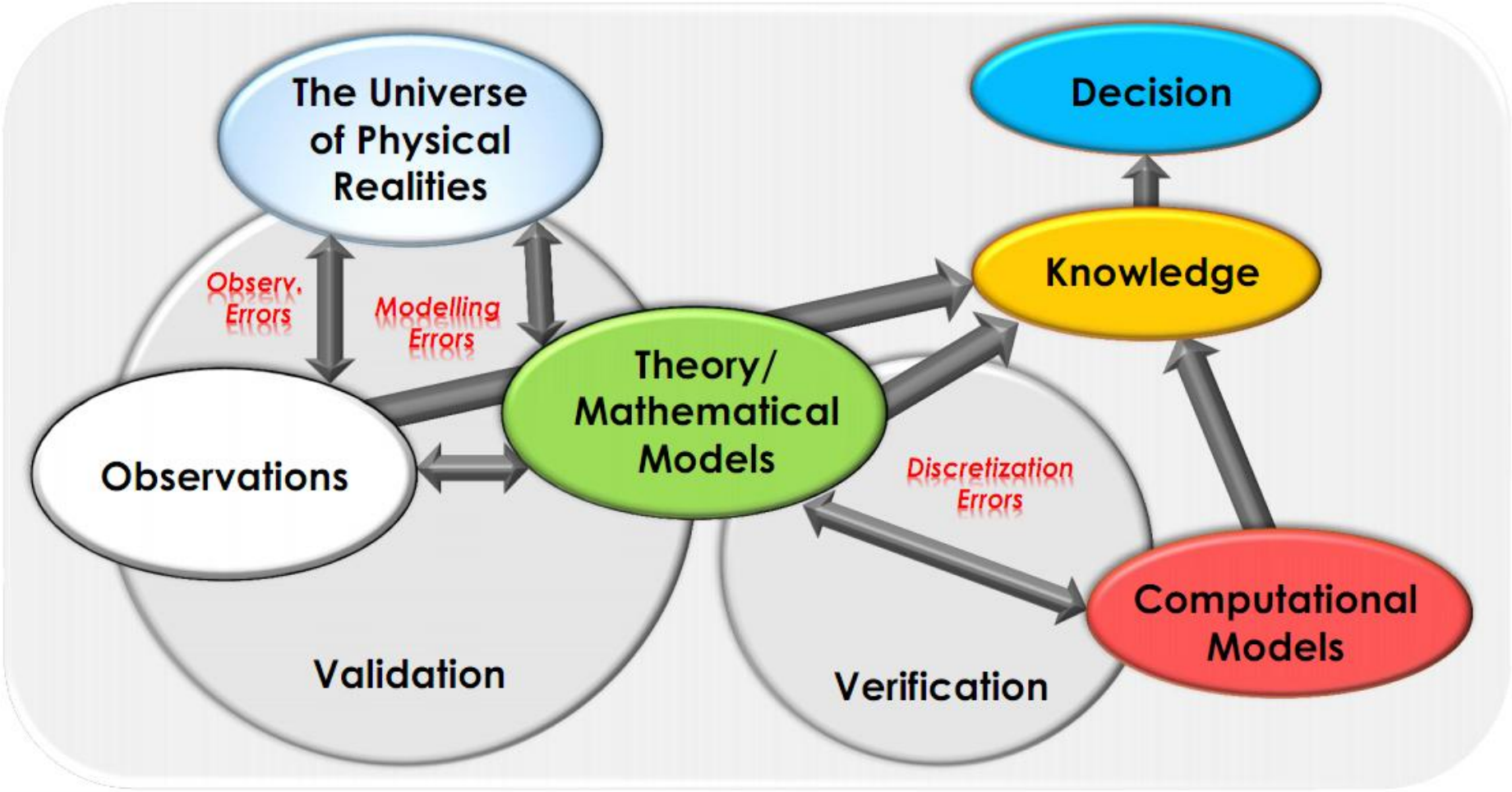




Initial Configuration

New Configuration after the Q optimization

And how certain we are?



Oden, Moser & Ghattas, "Computer Predictions with Quantified Uncertainty", SIAM NEWS, November 12, 2010.

So what can we do?

- Use Intuitionistic Fuzzy Sets for ***Q*** sets weights values calculation of the arcs ***A***: *Influence* – $\mu_A(x)$, *Dependence* – $\nu_A(x)$

- Create an IFS Map

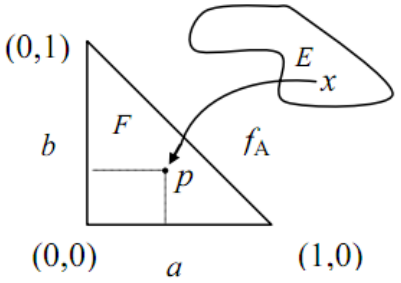
- Use IFS operators & experts opinion to move around the IFS Map

IFS Definition

According to Atanassov, 1999 IFS A^* in E is an object that has the following form:

(1) $$A^* = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in E\},$$

where: $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ are functions defined over an usual set A that defines the degrees of membership and non-membership of the element x from the fixed set E (called universe of disclosure) to A^*, shortly marked as A.



The sum of functions μ_A and ν_A values, is limited by the inequality:

(2) $$0 \le \mu_A(x) + \nu_A(x) \le 1$$

In accordance with the right constraint of (2), the function π_A that determines the degree of uncertainty, could be defined as follows:

(3) $$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x),$$

where: $\pi_A : E \to [0,1]$ is a function that determines the degree of uncertainty of x, $x \in E$.

*Atanassov, K. “Intutionistic Fuzzy Sets: Theory & Applications”, Springer - Verlag, 1999.

Some IFS Operators

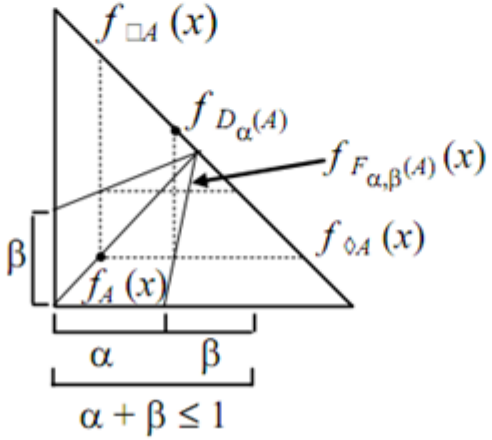
(4) $\Box A(x) = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle \mid x \in E\}$;

(5) $\Diamond A(x) = \{\langle x, \nu_A(x), 1 - \nu_A(x)\rangle \mid x \in E\}$;

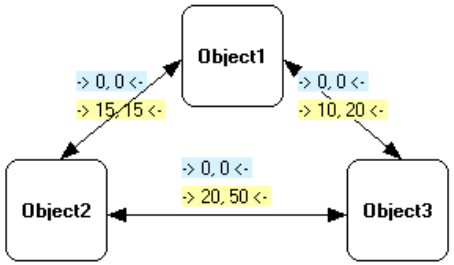
(6) $F_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + \beta.\pi_A(x)\rangle \mid x \in E\}$,

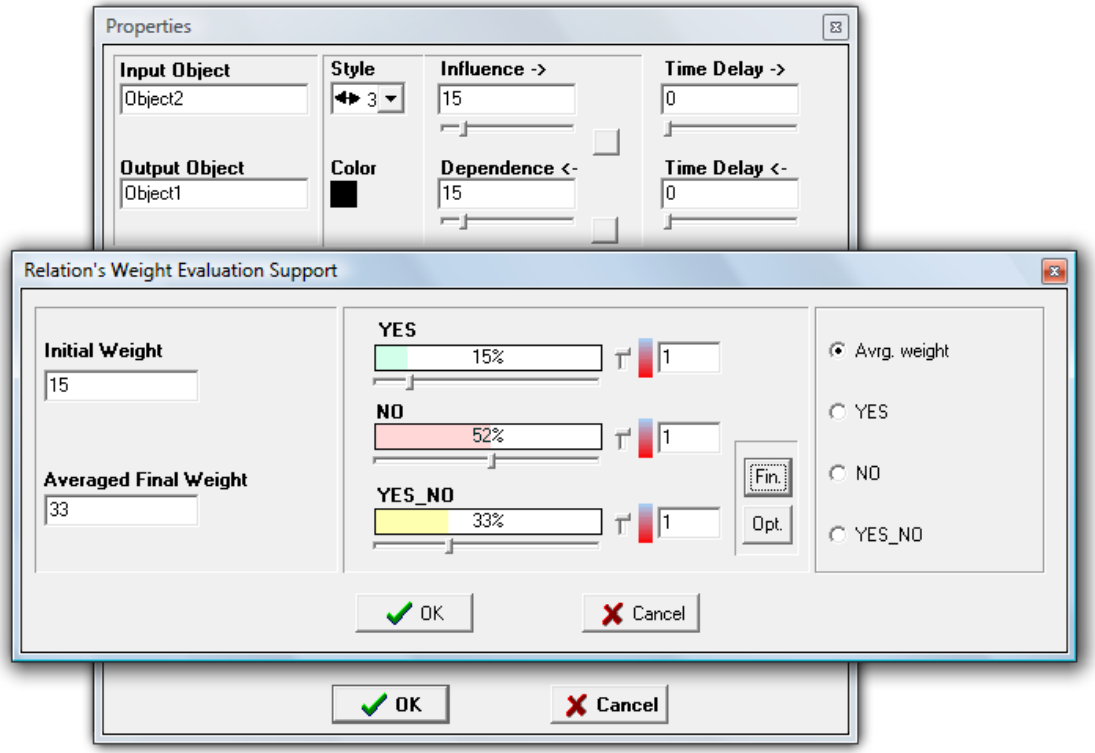
(7) $\alpha + \beta \leq 1, \alpha, \beta \in [0, 1]$,

where: $\Box$ is "necessity", $\Diamond$ is "possibility" (known from modal logic) and all other notations have the meaning discussed above (see IFS definition).



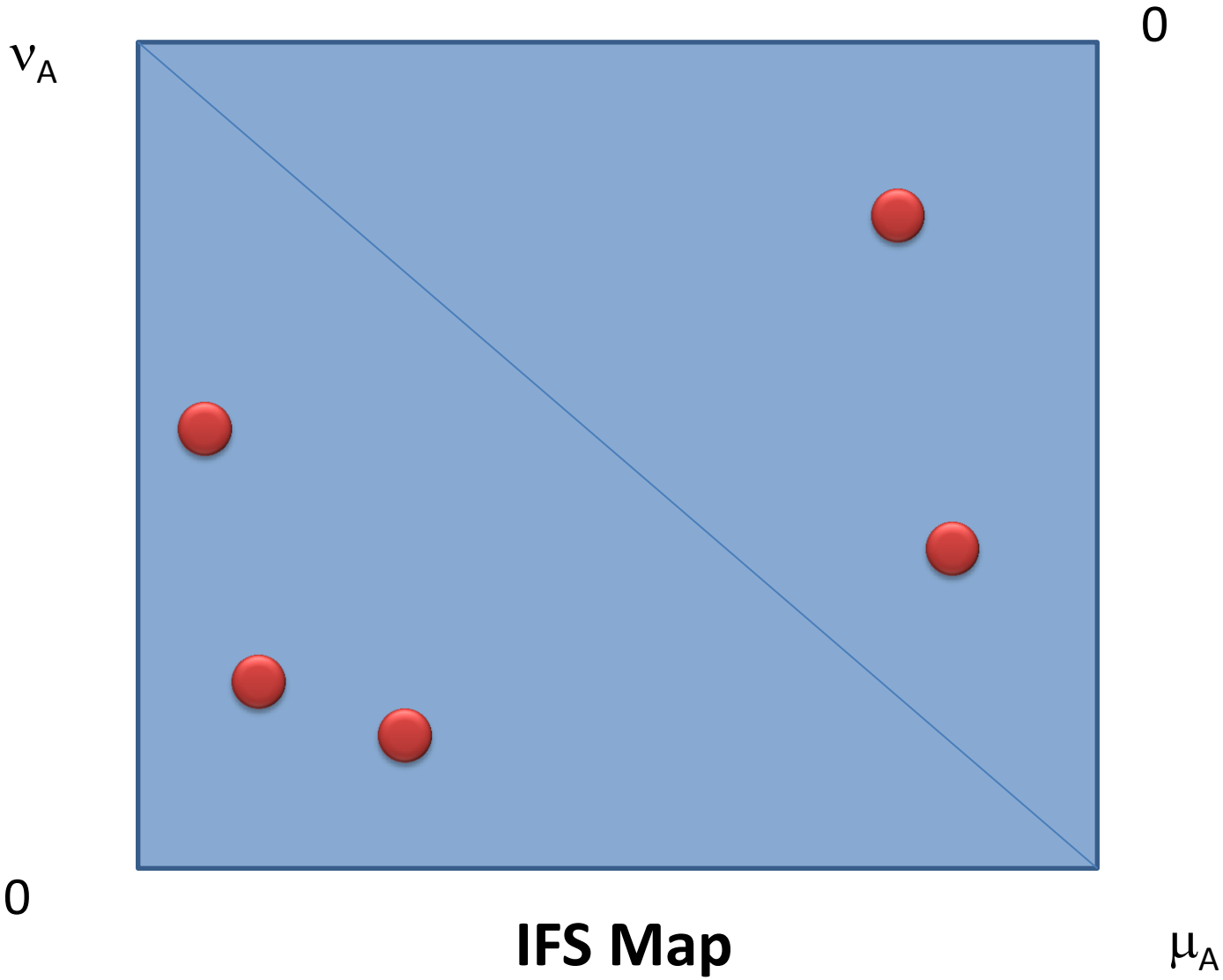
Software Implementation of IFS in I-SCIP* Environment





*Minchev, Z. “Intelligent Scenario Development for CAX”, IOS Press, 2007.

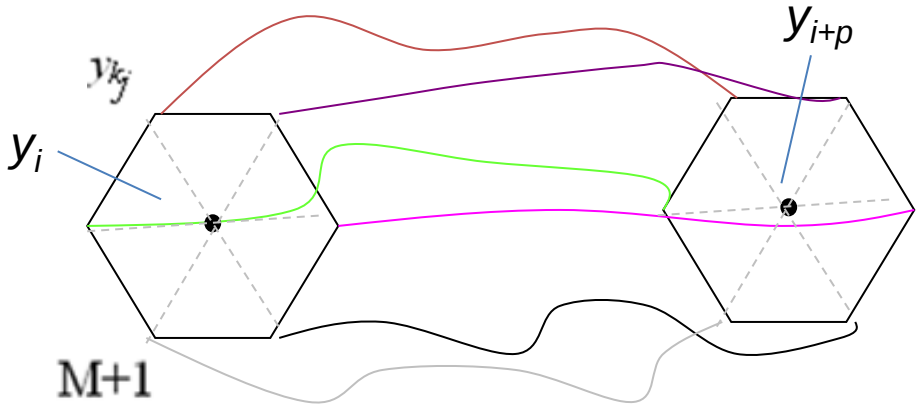
What to do more?



IFS Map

Observe the uncertainty dynamics of the objects in-between clusters transition

Approximate the solution of the inverse task of the one posed in Sugihara & May, 1990*, taking into account:



Nodes uncertainty dynamics example

$$y_{i+p} = \sum_{j=1}^{M+1} \bar{y}_{k_{j+p}} \; exp\,-\alpha\,[\,||\, y_i - y_{k_j} \,||\,]$$ Blinovska, K. & Malinovski, 1991

||.|| - Euclidean Distance in M dimension space;

y_{k_j} - j^{th} closest vector to y_i;

$\bar{y}_{k_{j+p}}$ - scalar value of the indexed $\bar{y}$ with k_j vector, p steps ahead

j =1,…,m+1; m-embedding dimension, M – current embedding dimension; M<=m

p – number of steps ahead;

α – real number

*Sugihara & May, “Nonlinear forecasting as a way of distinguishing chaos from measurement error in time series”, Nature 344, 734 - 741, 1990.

Conclusions

- The presented quadratic optimization application allows fast and easy movement in-between the cluster zones of the model vertices;

- What however is important to be included in this solution is to keep track of the uncertainty changes;

- A possible solution of the uncertainty changes could be obtained by utilization of IFS, experts opinion and IFS operators;

- An interesting future research work could be in the field of IFS map interpretation in 3D space and dynamic uncertainty correction.

Thank you for the attention!

Questions?